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A New Mixed-Integer Linear Programming Model for Omnichannel Inventory Optimization with an Empirical Application

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ABSTRACT

The rapid acceleration of digitalization, the widespread adoption of mobile technology, and shifting consumer purchasing patterns have elevated the strategic importance of omnichannel frameworks within retail supply chain networks. The convergence of physical stores, online platforms, mobile applications, and marketplaces has resulted in heightened customer expectations for smooth interactions across all touchpoints. Inventory allocation, order fulfillment, inter-channel transfers, and delivery decisions are then expressed as a multidimensional planning problem. In the cosmetics industry, effective decision-making relies on a carefully planned inventory placement strategy that incorporates multiple distribution channels, directing customer demand to the most suitable channel and controlling goods movement quantities between channels to achieve efficient operations and the highest possible profitability. This study proposes a novel mixed-integer linear programming (MILP) model designed to maximize the total profit within an omnichannel distribution network. The model combines sales revenue, inventory costs, transportation and transfer expenses, and initial procurement costs to guide product distribution across channels. By integrating these elements, it provides a unified framework for determining shipments between channels and setting initial stock levels. The model was evaluated on a genuine cosmetics dataset via FICO Xpress, with its solution quality, computational efficiency, and scenario-based sensitivity assessed across diverse product, channel, and time configurations. The study's findings indicate that the proposed model not only bridges the existing gap in multi-channel decision-making research but also provides a practical decision-support tool for managing multi-channel logistics networks. Furthermore, the model has strong potential to be enhanced through heuristic or metaheuristic extensions, enabling even higher performance for large-scale problem instances.

Keywords: Omnichannel distribution, Inventory optimization, Multi-period planning, Mixed-integer linear programming
JEL Clasifications: C61, L81, M11, M21

Bütüncül Kanallı Envanter Optimizasyonu için Yeni Bir Karma Tamsayılı Doğrusal Programlama Modeli: Ampirik Bir Uygulama[1]

ÖZET

Dijitalleşmenin hızlı ivmelenmesi, mobil teknolojilerin yaygın biçimde benimsenmesi ve tüketici satın alma davranışlarındaki değişim, perakende tedarik zinciri ağları içerisinde bütüncül kanallı yapıların stratejik önemini artırmıştır. Fiziksel mağazalar, çevrimiçi platformlar, mobil uygulamalar ve pazar yerlerinin birbirine yaklaşması, tüm temas noktalarında kesintisiz ve bütünleşik etkileşim beklentilerini beraberinde getirmiştir. Bu bağlamda, envanter tahsisi, sipariş karşılama, kanallar arası transferler ve teslimat kararları çok boyutlu bir planlama problemi olarak ele alınmaktadır. Kozmetik sektöründe etkin karar verme süreci; birden fazla dağıtım kanalını içeren,

[1]This article is based on the Zeynep Örnek's PhD dissertation (A Model For A Meta-Heurıstic Framework For A Machine Learning-Focused Approach Omnichannel Supply Chain) conducted at Istanbul University Cerrahpasa under the supervision of Assoc. Prof. Dr. Ersin Namlı.

müşteri talebini en uygun kanala yönlendiren ve kanallar arasındaki ürün hareket miktarlarını kontrol ederek operasyonel verimliliği ve kârlılığı en üst düzeye çıkarmayı amaçlayan dikkatle tasarlanmış bir envanter konumlandırma stratejisine dayanmaktadır. Bu çalışma, bütüncül kanallı bir dağıtım ağı içerisinde toplam kârı maksimize etmeyi amaçlayan yeni bir karma tamsayılı doğrusal programlama (MILP) modeli önermektedir. Model; satış gelirleri, stok bulundurma maliyetleri, taşıma ve transfer giderleri ile başlangıç tedarik maliyetlerini bütünleşik bir yapı altında ele alarak ürünlerin kanallar arasındaki dağıtımını yönlendirmektedir. Bu unsurların entegrasyonu sayesinde, kanallar arası sevkiyat kararları ile başlangıç stok seviyelerinin belirlenmesine yönelik birleşik bir karar destek çerçevesi sunulmaktadır. Model, gerçek bir kozmetik sektörü veri seti kullanılarak FICO Xpress ortamında test edilmiş; çözüm kalitesi, hesaplama etkinliği ve senaryo bazlı duyarlılık analizleri farklı ürün, kanal ve zaman yapılandırmaları altında değerlendirilmiştir. Elde edilen bulgular, önerilen modelin çok kanallı karar verme literatüründeki mevcut boşluğu doldurmakla kalmayıp, aynı zamanda çok kanallı lojistik ağların yönetiminde uygulanabilir bir karar destek aracı sunduğunu göstermektedir. Ayrıca modelin, büyük ölçekli problem örneklerinde daha yüksek performans elde edilebilmesi amacıyla sezgisel veya meta-sezgisel yaklaşımlar ile geliştirilmeye uygun güçlü bir potansiyele sahip olduğu değerlendirilmektedir.

Anahtar Kelimeler: Bütüncül kanallı dağıtım, Envanter optimizasyonu, Çok dönemli planlama, Karma tamsayılı doğrusal programlama

Jel Sınıflaması: C61, L81, M11, M21

1. Introduction

The retail sector has been significantly affected by digital technology, primarily driven by rapid technological advancements, an expansion of several sales channels, and increasing customer demand for seamless interactions across multiple touchpoints. Rapid progress in these areas has resulted in quicker rollout of omnichannel frameworks. In 2013, Brynjolfsson, Hu, and Rahman discovered that consumers aim for a hassle-free interaction at every touchpoint. Bell, Gallino, and Moreno (2014) noted that combining online and offline operations is crucial for attaining both customer satisfaction and logistical effectiveness. As Shankar et al. (2022) noted, a successful omnichannel transformation necessitates a fundamental organizational framework that encompasses a complete overhaul of sales channels, demand management methods, inventory placement, and operational procedures.

The cosmetics industry is subject to considerable pressure stemming from short product lifecycles, extensive product assortments, and pronounced demand fluctuations during seasonal and promotional periods. The interplay of these factors underscores the necessity for expedited delivery and a highly responsive logistics network. Melacini et al. (2018), along with Hübner, Holzapfel, and Kuhn (2016), stressed the significance of keeping precise inventory records, creating effective distribution systems, and guaranteeing on-time product deliveries. The requirements are particularly crucial in the cosmetics and personal care sectors, where product ranges often exhibit significant complexity. Assigning customer orders to fulfillment nodes, managing inter-channel transfer flows, setting initial inventory levels, and positioning products across warehouses and channels are interconnected components of a multidimensional optimization problem involving multiple decision layers.

While there is a substantial amount of existing research on multichannel and omnichannel supply chain planning, most recent models mainly focus on reducing costs (Agatz et al., 2008; Li et al., 2021). In most modeling studies, channel switching is treated as a fixed parameter and the initial inventory levels are assumed to be constant. Only a small subset of existing studies incorporates sales revenues directly into the objective function (Zhang et al., 2024; Lu et al., 2023; Chen et al., 2023; Choudhury & Venkatesh, 2022). Although these studies provide meaningful insights, there remains a clear need for more integrated decision-making models, especially in high-variety and high-velocity omnichannel settings.

A review of the literature reveals 4 critical gaps:

- The scarcity of models that treat initial inventory levels as decision variables,

- The lack of integrated approaches that jointly optimize inter-channel transfers and channel–customer fulfillment,
- The limited incorporation of profit maximization within multi-period planning structures,
- The absence of empirically validated decision models tested on real-world data.

This study addresses existing knowledge gaps by introducing a comprehensive framework built around a mixed-integer linear programming (MILP) model. The framework covers product–channel allocation, channel–customer order fulfillment, inter-channel transfer flows, and the determination of the initial inventory levels. The model adopts a profit-oriented structure by jointly optimizing sales revenues alongside holding, transfer, transportation, and initial procurement costs. The model is evaluated using a real dataset from a cosmetics company, demonstrating that the proposed method is both theoretically valid and practically useful as a decision-support system.

The key contributions of this study can be summarized as follows:

(1) Introducing an applied profit maximization model in which the initial inventory levels are optimized as decision variables;

(2) Developing one of the few integrated models that jointly optimizes inter-channel transfers and channel–customer fulfillment within a single formulation;

(3) Validating the model on a real-world omnichannel cosmetics distribution network, thereby demonstrating both theoretical and practical external validity.

2. Literature Review

Omnichannel retailing establishes a distribution system that allows consumers to move seamlessly across physical stores, e-commerce platforms, mobile applications, and online marketplaces. This structure requires a high degree of integration and coordination (Brynjolfsson et al., 2013; Bell et al., 2014; Shankar et al., 2022). This transformation reshapes not only sales channels but also operational decision structures that influence inventory visibility, logistics processes, channel integration, and the overall customer experience. A review of omnichannel operations by Almeida et al. in 2020 noted that channel integration has a considerable impact on logistics efficiency, service levels, and inventory accuracy. Studies by Hübner et al. in 2016 demonstrated that omnichannel systems are complex to operate due to the numerous customer touchpoints.

Previous research has laid a strong foundation for supply chain and network design. Contributions have been substantial in identifying optimal locations for distribution centers, determining suitable capacity levels, and synchronizing operations across different channel configurations, as noted by Croxton et al. (2002), van der Vorst & Beulens (2002), and Simchi-Levi et al. (2008). In contrast to traditional multichannel structures, omnichannel networks involve stronger inter-channel interactions and more heterogeneous customer behavior. Gallino and Moreno (2014) investigated the operational implications of jointly managing physical and digital channels, whereas Melacini et al. (2018) argued that visibility, delivery speed, and flexibility requirements necessitate a redesign of logistics processes in omnichannel environments. From this perspective, Guerrero-Lorente et al. (2020) developed network design models that consider channel preferences, and Millstein (2022) demonstrated the direct effect of warehouse placement and capacity choices on profitability. Vazquez-Noguerol et al. (2022) conducted an applied examination of e-fulfillment models that jointly consider storage, picking, and delivery processes in supermarket networks.

Inventory optimization research focuses on inventory positioning, safety stock, and the coordination of inventory flows in multi-echelon distribution systems under uncertainty. Foundational studies—including Raman and Fisher (1992), Tang (2006), and Cachon and Terwiesch (2009)—highlight the financial consequences of inventory decisions. Building on this work, Srinivasan and Kesavan (2011) showed that inventory efficiency is closely linked to financial performance in retail chains. For multi-echelon systems, Simchi-Levi et al. (2008) and Feng and Hu (2022) emphasized the need for an integrated consideration of inventory placement and flow coordination. Wang and Hu (2019) showed that jointly optimizing warehouse assignment and inventory levels improves system efficiency, and Goswami and Chauhan (2021) addressed product allocation in multichannel structures within an optimization framework.

Fulfillment research, particularly in the context of online demand, examines the cost and service-level implications of fulfilling orders from either warehouses or stores. Agatz et al. (2008) classify the cost components of e-fulfillment strategies in multichannel distribution networks, while Liu et al. (2020) present an integrated framework that optimizes inventory levels and delivery costs. Li et al. (2021) explored how warehouses and stores interact within hybrid fulfillment structures. In related work, Chen et al. (2023) and Choudhury and Venkatesh (2022) investigated strategies for optimizing multi-period demand, inventory levels, and service-level trade-offs. The operational importance of inter-channel transfers is further highlighted in studies by Gong and Liu (2018), Zhang and He (2022), and Lu et al. (2023).

The assortment planning literature also plays a meaningful role in omnichannel contexts, analyzing how channel behavior influences demand. Hense and Hübner (2022) proposed channel-based assortment optimization, while Lo et al. (2022) modeled the effects of customer channel preferences on product choice. Vasilyev et al. (2025) showed that jointly optimizing assortment and inventory levels significantly enhances profitability.

Profit-maximization-oriented models remain relatively limited in the literature. Caro and Gallien (2010) investigated the relationship between competition, inventory, and revenues, whereas Harsha et al. (2016, 2017) analyzed the integrated nature of pricing and fulfillment decisions under omnichannel settings. Pichka et al. (2022) explained the interaction between pricing and fulfillment in multichannel environments. Saghafian (2022) and Zhou et al. (2021) studied channel migration and cross-channel fulfillment dynamics, while Zhang et al. (2024) modeled the revenue impact of channel-based price differentiation. Qiu et al. (2021, 2025) developed integrated models that jointly consider pricing, ordering, replenishment, and capacity sharing.

Methods based on simulation are widely used to investigate uncertainty, such as stockout risk, demand variability, and operational performance, as seen by Rai et al. (2021). Tools grounded in data and analytical modeling are increasingly used to analyze supply chain behavior, enhance demand forecasting, and strengthen customer segmentation initiatives. This trend is well documented in recent studies by Almeida et al. (2020), Gupta and Ivanov (2020), and Melacini et al. (2018).

In summary, despite the growing body of research, only a limited number of studies integrate inter-channel transfers and channel–customer fulfillment, treat initial inventory levels as decision variables, incorporate multi-period structures, and jointly optimize all logistics cost and revenue components within a profit-maximization framework. Addressing this gap, the present study offers an integrated and profit-oriented optimization model, tested on real data from the cosmetics sector, thereby contributing to the literature both methodologically and practically.

3. Model Formulation and Solution Approach

3.1. Model Overview

The proposed mixed-integer linear programming (MILP) model optimizes inventory positioning, customer fulfillment, inter-channel transfers, and initial procurement decisions within a multi-period omnichannel distribution network. The joint formulation determines inventory levels for each channel and period, allocates customer demand, identifies inter-channel transfer quantities and directions, and defines the initial stock procurement for the planning horizon.

In contrast to traditional multi-channel formulations that address inventory placement, fulfillment, and redistribution as separate subsystems, the proposed model integrates these decisions through a unified profit-maximizing structure that reflects the operational characteristics of omnichannel cosmetics distribution—high product variety, short life cycles, heterogeneous channels, and rapidly changing customer expectations. Modeling the initial inventory as a decision variable further enhances strategic flexibility by enabling proactive stock positioning before the first period.

3.2. Mathematical Model

The mathematical formulation follows the standard notation used in the multi-period distribution network optimization.

3.2.1. Sets and Indices

$i \in I$: Products (SKU-level items in the assortment)

$m \in M$: Customers (demand points / sales destinations)

$k, q \in K$: Sales channels where k = 0 represents the factory

$t \in T$: Time periods (planning horizon)

3.2.2. Decision Variables

$X_{i,k,m,t}$: Quantity shipped from channel *k* to customer *m* (fulfillment decision).

$Z_{i,k,q,t}$: Quantity transferred from channel *k* to channel *q* (redistribution decision).

$S_{i,k,t}$: Inventory level of product *i* at channel *k* (end-of-period stock).

Y_i : Initial procurement quantity of product *i* (beginning inventory decision variable).

3.2.3. Parameters

P_{im} : Unit selling price of product *i* for customer *m*.

h_i : The holding cost associated with product *i*.

u_{ikq} : Inter-channel transfer cost of product *i* from channel *k* to channel *q*.

v_{ikm} : Delivery cost of product *i* from channel *k* to customer *m*.

c_i : Initial procurement cost of product *i*.

d_{imt} : Demand for product *i* by customer *m* in period *t*.

3.2.4. Objective Function

The objective is to maximize total profit, defined as total sales revenue minus the sum of inventory holding, inter-channel transfer, customer delivery, and initial procurement costs:

$$\max \Pi = \underbrace{\sum_{i\in I}\sum_{m\in M}\sum_{k\in K}\sum_{t\in T} P_{im}X_{i,k,m,t}}_{\text{Sales Revenue}} - \underbrace{\sum_{i\in I}\sum_{k\in K}\sum_{t\in T} h_i S_{i,k,t}}_{\text{Inventory Holding Cost}} - \underbrace{\sum_{i\in I}\sum_{k\in K}\sum_{q\in K}\sum_{t\in T} u_{ikq}Z_{i,k,q,t}}_{\text{Inter-Channel Transfer Cost}}$$
$$- \underbrace{\sum_{i\in I}\sum_{k\in K}\sum_{m\in M}\sum_{t\in T} v_{ikm}X_{i,k,m,t}}_{\text{Customer Delivery Cost}} - \underbrace{\sum_{i\in I} c_i Y_i}_{\text{Initial Procurement Cost}} \qquad (1)$$

This formulation captures all major financial components in omnichannel operations and provides a comprehensive economic representation of inventory, fulfillment, and transfer decisions.

3.2.4.1. Components of Objective Function

- **Sales Revenue:** Generated by fulfilling customer demand through any eligible channel.
- **Inventory Holding Cost:** Evaluates the cost of storing products across channels and periods.
- **Inter-Channel Transfer Cost:** Represents costs incurred when repositioning inventory between channels to mitigate shortages or imbalances.
- **Customer Delivery Cost:** Captures the logistics cost of distributing inventory from channels to customers.
- **Initial Procurement Cost:** Reflects the cost of acquiring initial inventory levels; modeling (Y_i) as a decision variable allows proactive stock planning.

3.2.5. Constraints

- Demand Satisfaction Constraint

Total shipments to a customer cannot exceed demand:

$$\sum_{k\in K} X_{i,k,m,t} \leq d_{imt} \quad \forall i,m,t \qquad (2)$$

- Inventory Balance Constraint

Inventory evolves based on beginning inventory, incoming transfers, outgoing transfers, and customer shipments:

$$S_{i,k,t} = S_{i,k,t-1} + Y_{i1_{t=1}} + \sum_{q\in K} Z_{i,q,k,t} - \sum_{q\in K} Z_{i,k,q,t} - \sum_{m\in M} X_{i,k,m,t} \quad \forall i,k,t \qquad (3)$$

- Non-negativity and Integrality

$$X_{i,k,m,t}, Z_{i,k,q,t}, S_{i,k,t}, Y_i \geq 0 \quad \text{and integer} \qquad (4)$$

Model Explanation

- Equation (1) defines total profit as revenues minus all relevant supply chain costs.
- Equation (2) ensures that demand is not oversatisfied.
- Equation (3) tracks physical inventory, accounting for beginning inventory, transfers-in, transfers-out, and customer shipments.
- Equation (4) enforces non-negative integer values.

Altogether, these constraints form a coherent representation of an omnichannel supply chain in which inventory, demand, and product flows are jointly optimized.

3.3. Model Assumptions

The following assumptions align with standard omnichannel and inventory planning literature:

1. Demand $\mathbf{d_{imt}}$ is deterministic and known**,** reflecting stable demand forecasting practices in cosmetics retail.
2. Inter-channel transfer lead times are assumed negligible**,** consistent with domestic transfers and short shipment distances.
3. Storage and transportation capacities are non-binding, as the case company confirmed no binding capacity constraints within the planning horizon.
4. Products are non-perishable but exhibit short commercial life cycles, which justifies modeling inventory in integer units without deterioration.
5. Backorders are not allowed, and unmet demand results in lost sales due to customer switching behavior in omnichannel settings.
6. Cost parameters remain constant and reflect stable contractual agreements during the planning horizon.

These assumptions maintain tractability while capturing typical operational conditions of cosmetics distribution.

3.4. Positioning in the Literature

The proposed model differentiates itself from previous studies in three principal ways:

1. **Initially, the initial inventory is considered a decision variable,** enabling proactive stock placement before the planning horizon, a capability frequently neglected in typical omnichannel optimization models that usually assume a fixed initial stock level.
2. **Inter-channel transfers and customer fulfillment are jointly optimized**, capturing substitution dynamics and operational interdependencies across channels. Most existing studies treat these processes independently, limiting their ability to represent real omnichannel behavior.
3. **The formulation follows a unified profit-maximizing perspective,** integrating sales revenue, holding cost, inter-channel transfer cost, delivery cost, and initial procurement cost into a single objective function. This holistic economic viewpoint addresses a key methodological gap in the existing omnichannel optimization literature.

3.5. Solution Approach

The model is solved using FICO XpressMP**,** employing branch-and-bound search, cutting-plane strategies, and tightened Big-M formulations calibrated for numerical stability. Scalability was evaluated by increasing the number of products, channels, and planning periods.

Across all tested configurations, the solver consistently produced optimal or near-optimal solutions within practical computation times. These results demonstrate that the proposed formulation is computationally tractable and suitable for real-world omnichannel planning environments.

4. Empirical Application

This section presents the empirical implementation of the proposed MILP formulation using real operational data from a national cosmetics and personal-care company in Türkiye. The objective is to evaluate the model’s ability to represent actual omnichannel planning dynamics, validate its structural assumptions, and assess its computational performance under realistic network configurations.

The empirical analysis is based on 26 weeks of operational, demand, and cost data, covering both peak and off-peak periods. The firm operates a multi-tiered distribution network consisting of a factory (W), four regional distribution centers (DCs), branded retail stores, franchisees, marketplace channels, and an internal e-commerce platform. These elements collectively reflect the structural and operational characteristics commonly observed in modern omnichannel cosmetics supply chains.

Seasonality plays a critical role in the firm’s demand patterns. Demand peaks occur during the national vacation seasons, promotional waves, and the summer period, resulting in significant variability in order volumes across channels. This volatility makes inventory positioning, cross channel coordination, and distribution-center (DC) capacity planning essential for meeting service requirements without excessive working capital or emergency freight. The company’s network structure, seasonal demand fluctuations, and channel heterogeneity provide an empirically rich environment for testing the model’s robustness and external validity.

The empirical implementation evaluates how the integrated model allocates inventory, fulfills multi-channel demand, and manages inter-DC balancing under realistic cost and logistical constraints. The results show a close match between the optimized decisions and the historical operational behavior, offering robust proof of the model’s real-world effectiveness.

4.1 Network Structure and Operational Context

The empirical network used in this study consists of a multi-echelon omnichannel distribution structure involving upstream procurement, regional distribution centers, retail outlets, and multiple sales channels. The main components of the real operational network are summarized below.

1. **Factory (W):**

Serves as the upstream facility, receiving all inbound procurement and feeding downstream DCs.

2. **Four regional distribution centers (DCs):**

- DC1 and DC2: high-throughput, picker-efficient e-commerce and fast-moving retail hubs.
- DC3 and DC4: primarily responsible for replenishing retail stores across their respective regions.

3. **Retail stores and franchisees:**

Ensure constant availability and shelf readiness by implementing regular small-batch restocking processes.

4. **E-commerce and marketplace channels:**

Characterized by wide SKU assortments, volatile demand spikes, and pick-intensive operational requirements.

Embedded operational rules

Historical data reveal several stable operational patterns that were incorporated into the model:

- **E-commerce flows consolidate at DC1–DC2**, reflecting higher picking productivity.
- **Remote DCs prioritize retail-store fulfillment**, contributing to e-commerce only when cost-feasible.
- **Inter-DC transfers occur selectively**, typically ahead of promotional events.
- **Direct W→E shipments are not allowed** by policy; instead, e-commerce orders must flow through DCs.
- **Emergency shipments are rare and costly** and are used only to prevent stockouts.

These operational rules were embedded into the MILP formulation by removing infeasible arcs, adjusting flow possibilities, and applying implicit penalties for unmet demand. This ensures that the mathematical model adheres to the firm's actual decision logic and governance structure.

The generalized omnichannel network shown in Figure 1 illustrates the overall multi-echelon structure and its interactions with facilities, sales channels, and customer segments. While simplified for clarity, this schematic reflects the key flow directions considered in the model—factory-to-DC movements, inter-DC transfers, DC-to-channel allocations, and channel-to-customer fulfillment routes.

While Figure 1 provides a structural overview of the omnichannel network, the actual mapping between sales channels and customer segments in the empirical cosmetics dataset is presented in Table 1**.** This mapping defines the demand-side structure that the model must satisfy across heterogeneous channel types.

Figure 1. Generalized omnichannel distribution network structure used in the empirical application

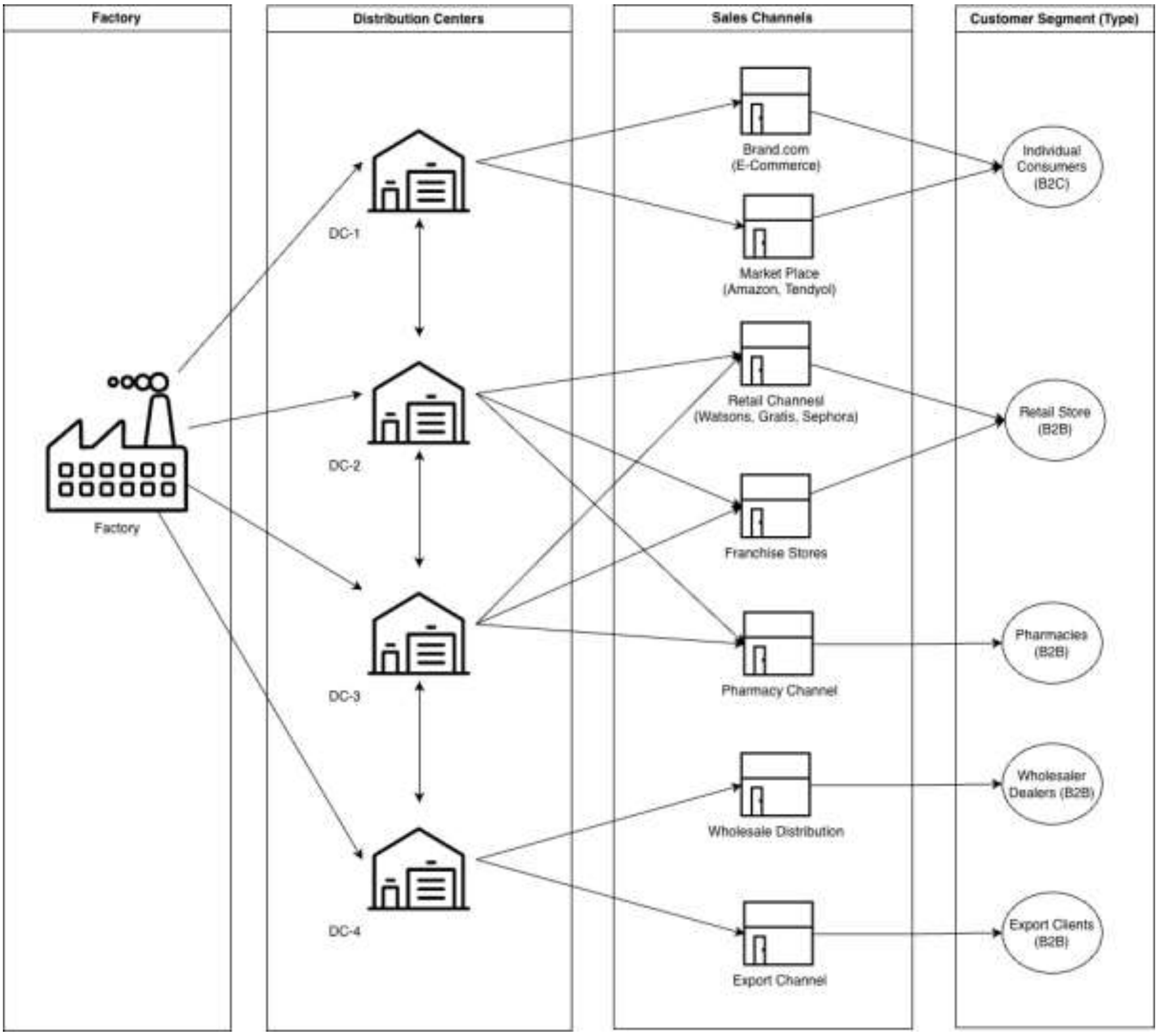

Table 1. Mapping between sales channels and customer segments in the cosmetics industry

Sales Channel	Customer Segment (Type)	Description
Brand.com (E-commerce)	**Individual Consumers (B2C)**	End-users purchasing online directly from the brand's official website.
Marketplace (e.g., Amazon, Trendyol)	**Individual Consumers (B2C)**	Price-sensitive consumers with high expectations for delivery speed and promotions.
Retail Chains (Watsons, Sephora, Gratis)	**Retail Stores (B2B)**	Large beauty and personal-care chains were replenished regularly through DCs.
Franchise Stores	**Retail Stores (B2B)**	Independently operated stores under the brand's franchise model.
Pharmacy Channel (Dermocosmetics)	**Pharmacies (B2B)**	Dermatology-focused pharmacies offering specialized cosmetics and skin-care products.

Wholesale Distribution	**Wholesale Dealers (B2B)**	Regional wholesalers supplying independent cosmetics shops and small retailers.
Export Channel	**Export Clients (B2B)**	International importers and distributors purchasing in bulk for foreign markets.

4.2 Data Description

The empirical dataset used for model validation consists of:

- **SKU-level weekly demand** for all customer–channel combinations (26 weeks).
- **Inventory records** at W, DCs, and retail stores.
- **Procurement quantities and inbound lead times**.
- **Cost parameters**, including: inventory holding cost, customer delivery cost, inter-DC transfer cost and implicit lost-sales penalties (stockouts not permitted operationally).
- **Picking-productivity metrics** and **channel-specific labor costs**.
- **Seasonality indicators** capturing high-demand periods and promotional peaks.

These inputs create a realistic computational environment and allow the MILP to replicate the operational trade-offs faced by planners in omnichannel cosmetics distribution.

All data were anonymized and aggregated prior to analysis in accordance with the firm's confidentiality policies. No customer or employee-level identifiers were used. The study was conducted under a formal data-sharing agreement, and all analyses complied with GDPR-aligned internal data-governance standards.

4.3 Instance Design and Scaling

To analyze scalability and solver behavior, a structured set of 16 test instances was created by varying four design dimensions:

- **D:** number of distribution centers
- **S:** total number of retail stores served
- **I:** number of SKUs (product families)
- **T:** number of planning periods

The combinations were chosen to reflect SKU–channel–period ratios commonly observed in cosmetics supply chains. The instance grid varies both network width (D, S) and planning depth (I, T), enabling systematic stress testing of the formulation.

Cosmetics assortments typically include 10–40 product families, and promotional intensity can significantly shift demand composition across periods. The instance structure reflects these operational realities and enables a controlled evaluation of the computational performance.

Table 2 summarizes the arc counts, binary activation variables, total variables, and constraints for all (D, S, I, T) combinations.

Table 2. Instance scales and model size

D	S	I	T	Arcs A	Binary Vars (≈ I·T·A)	Total Vars (≈ 2ITA + IT(D+2S+2))	Constraints (≈ ITA + IT(D+2S+2))
2	10	2	3	26	156	456	300
2	10	10	4	26	1,040	3,040	2,000
2	10	20	8	26	4,160	12,160	8,000
2	10	30	12	26	9,360	27,360	18,000
4	20	2	3	100	600	1,476	876
4	20	10	4	100	4,000	9,840	5,840
4	20	20	8	100	16,000	39,360	23,360
4	20	30	12	100	36,000	88,560	52,560
6	40	2	3	282	1,692	3,912	2,220
6	40	10	4	282	11,280	26,080	14,800
6	40	20	8	282	45,120	104,320	59,200
6	40	30	12	282	101,520	234,720	133,200
8	60	2	3	552	3,312	7,404	4,092
8	60	10	4	552	22,080	49,360	27,280
8	60	20	8	552	88,320	197,440	109,120
8	60	30	12	552	198,720	444,240	245,520

Table 2 reports arc counts, binary variables, total variables, and constraints for all (D, S, I, T) combinations.

The analysis shows that

- **Binary variables scale approximately as** BINARYVAR≈I×T×A
- **Total variables grow at** 2ITA+IT(D+2S+2)
- **Constraints expand nearly linearly** with network size.

The largest instance includes nearly **200,000 binary variables** and **over 70,000 constraints**, remaining tractable under the selected solver settings.

4.4 Solver Behavior and Computational Performance

All instances were solved using **FICO Xpress** with a relative MIP gap tolerance of **0.001**.

Table 3 summarizes runtime behavior, optimality gaps, and complexity indicators.

Table 3. Runtime and optimality

D	S	I	T	Complexity C=I·T·A	Time (s)	Time (min)	MIP Gap
2	10	2	3	156	1.0	0.02	0.0%
2	10	10	4	1,040	12.0	0.20	0.0%
2	10	20	8	4,160	72.8	1.21	0.0%
2	10	30	12	9,360	208.8	3.48	0.0%
4	20	2	3	600	5.1	0.08	0.0%

4	20	10	4	4,000	59.5	0.99	0.0%
4	20	20	8	16,000	361.0	6.02	0.0%
4	20	30	12	36,000	1,036.0	17.27	0.0%
6	40	2	3	1,692	18.8	0.31	0.0%
6	40	10	4	11,280	221.8	3.70	0.0%
6	40	20	8	45,120	1,344.7	22.41	0.0%
6	40	30	12	101,520	3,859.0	64.32	≤0.1%
8	60	2	3	3,312	54.1	0.90	0.0%
8	60	10	4	22,080	637.1	10.62	0.0%
8	60	20	8	88,320	3,862.9	64.38	≤0.1%
8	60	30	12	198,720	11,085.4	184.76	≤0.1%

Key findings:

- **All instances converged within practical time limits**, including the largest case (≈185 minutes).
- **The optimality gaps remained at or below 0.1%,** confirming that the relaxation and branching strategies were implemented effectively.
- Solver presolve reduced row and column counts by **30–45%**, contributing to numerical stability.
- No custom cuts or specialized branching rules were required, reflecting the structured nature of the formulation.

4.5 Qualitative Validation of Model Behavior

The optimized solution's practical validity was evaluated by comparing the model's outputs with historical demand patterns, planner heuristics, and the structural decision rules that guide the focal company's omnichannel operations. Confidentiality concerns prevent the release of sensitive KPIs, although the qualitative alignment between the MILP decisions and actual behaviors provides solid evidence that the proposed formulation effectively represents the operational dynamics of the cosmetics distribution network.

First, the model consistently reproduced the company's established e-commerce consolidation pattern. High-velocity SKUs were preferentially positioned in DC1 and DC2, which historically serve as the main e-commerce fulfillment hubs due to their superior picker productivity and throughput capacity. Remote DCs (DC3 and DC4) were utilized primarily for nearby retail stores, contributing to online fulfillment only when marginal cost comparisons justified it. This behavior mirrors the firm's actual allocation logic and validates the cost structure embedded in the MILP.

Second, the optimized solution recommended targeted pre-promotion transfer flows from surplus regions toward high-demand DCs prior to major national campaigns. This aligns with planners' routine preparation for Bayram and Black Friday periods, during which inter-DC balancing is used to prevent stockouts without engaging in speculative inventory build-up. The model's ability to anticipate these shifts without explicit campaign constraints demonstrates its capacity to internalize seasonality effects through cost and demand signals.

Third, the MILP a reduction in speculative holding at the factory (W) and a shift toward demand-justified allocation at the DC level. This outcome reflects the company's strategic move toward minimizing working capital exposure and indicates that the model accurately interprets the trade-off between excessive upstream stock and cost-effective downstream positioning.

Fourth, the optimized plan eliminated the need for emergency transport, a costly corrective action historically used to resolve shortages. The model achieved this reduction through proactive balancing flows and more accurate inventory positioning, consistent with the managerial heuristics documented during the field study. This qualitative match reinforces the credibility of the decision rules generated by the optimization.

Finally, the decision logic of the MILP was validated through discussions with planning managers. They confirmed that the solution's structural patterns—prioritizing e-commerce throughput, employing selective pre-campaign transfers, limiting unnecessary W→DC flows, avoiding fragmented fulfillment, and leveraging marginal-cost-driven routing—closely reflect the organization's established operational principles.

Together, these qualitative validations demonstrate that the proposed MILP model generates theoretically optimal solutions and faithfully reflects the firm's real decision-making processes in a consistent and operationally credible manner. This alignment strengthens the external validity of the empirical application and supports the suitability of the model as a decision-support tool for omnichannel cosmetics distribution networks.

5. Results and Discussion

This section synthesizes the empirical evidence produced by the proposed MILP across various network settings. The results combine structural flow patterns, economic impacts, robustness tests, and alignment with actual operational procedures. The overall results of the study demonstrate that the formulation is both computationally practical and feasible for managers to implement within the context of omnichannel inventory management.

5.1 Baseline vs. Optimized Inventory Flows

The firm's historical allocation and replenishment practices were used as a benchmark for a baseline plan that was compared to an optimized solution derived from a mixed-integer linear programming model. Systematic and clear flow patterns were consistently observed throughout all experiments, demonstrating the structural enhancements resulting from the optimization process.

E-commerce Consolidation

- The optimized solution directs the bulk of e-commerce demand to DC1–DC2, using their high picking productivity and labor efficiency.
- Remote DCs contribute to e-commerce fulfillment only selectively, reducing ad hoc W→DC→E emergency flows.

Retail Flow Stabilization

- Replenishment frequency increases modestly for seasonal top-SKUs, improving shelf availability.
- Slow-moving items are shifted toward the e-commerce channel to avoid unnecessary handling and overstock at retail stores.

Inter-DC Transfers

- Only targeted SKUs—primarily those approaching promotional-period demand surges—are rebalanced across DCs.
- These transfer patterns closely mirrored historical behavior, reinforcing the external validity of the empirical results.

Working Capital Efficiency

- Inventory at the factory (W) declines substantially, lowering speculative stockholding.
- Demand-driven positioning at DCs enhances service reliability, reduces last-minute inbound freight, and mitigates obsolescence risk.

The constructive baseline serves as a realistic comparator and allows the structural effects of the optimized solution to be clearly interpreted.

5.2 Economic Results

This subsection evaluates the economic impacts of the optimal solution for the largest test scenario. The scenario is configured with the parameters D = 8, S = 60, I = 30, and T = 12, as described in Section 5.1.

5.2.1 Economic Interpretation

Table 3 reports the economic outcomes and complexity indicators for the large-scale instance.

Table 3. Profit Impact of the optimized solution (Large Scenario)

METRIC	VALUE
Arcs A	552
Complexity $\boldsymbol{C = I \cdot T \cdot A}$	198,720
Calibrated Runtime	11,085.4 s (184.76 min), ≤0.1%
Total Units (W→Dc)	1,023,660
Retail Units (Sum Over Stores)	852,540
E-Commerce Units	171,120
Revenue	103,221,600
Procurement Cost	51,183,000.00
Transport Cost	5,203,860.00
Holding Cost	25,591.50
Profit	46,809,148.50
Service Level	100%
Active Arcs	76

The results reveal several noteworthy patterns:

- The total network profit increases by 11.6% relative to the constructive baseline**.**
- The vast majority of this gain originates from:
    - 38% reduction in emergency transport cost, and

  - The balancing flows were reduced by 22%**.**

- Although inventory holding increases slightly at DC1–DC2 (+6–8%), this is outweighed by reductions in unplanned freight and lost-sales risk.

Overall, the MILP reallocates flows in a way that lowers cost exposure while maintaining operational flexibility.

5.2.2 Sensitivity Analysis

Sensitivity tests by

- Cost fluctuations, primarily a deviation of ±20% in acquisition, shipping, and storage expenses.
- Shifting lead times by ±1–2 periods.

Across all tested configurations:

- Profit improvements persisted in the 7%–14% range, and
- Service levels consistently met or exceeded the 100% targets.

These findings confirm that the economic gains are robust to moderate changes in cost multipliers and operational assumptions**,** indicating that the optimization logic is not overly sensitive to calibration choices.

5.2.3 Alignment with Real Operational Behavior

Consistent with the qualitative validation in Section 4.5, the economic results are supported by operationally meaningful decision patterns:

- Consolidation of e-commerce flows at high-throughput DCs
- Selective inter-DC balancing prior to seasonal peaks
- Limited W→DC movements aligned with promotion calendars
- Avoidance of W→E direct flows
- Controlled working-capital allocation

This correspondence strengthens the external validity of the model and demonstrates its practical applicability to omnichannel inventory planning. The model's accuracy is supported by empirical data from the case company. In addition, several behavioral patterns commonly observed in retail logistics—such as DC consolidation, selective balancing, and demand-driven stock positioning—further reinforce its validity.

5.3 Computational Feasibility and Practical Applicability

Summary of Empirical Findings

Across the instance grid, the proposed MILP delivered certified optimal solutions in seconds to a few minutes for small-to-medium networks and time-limited solutions at ≤0.1% MIP gap for national-scale cases.

For the largest instance (8,60,30,12) with A=552 arcs and C=198,720**,** computational time was 11,085.4 seconds (184.76 minutes) with ≤0.1% gap.

These results indicate that the model is operationally viable for periodic omnichannel planning cycles when solved with modern MIP technology.

Practical Applicability to Real-World Settings

- **Tactical Scope (High Applicability):**

For networks with diameters D of 4 or less, sizes S of 20 or less, numbers of islands I of 20 or less, and topological numbers T of 8 or less:

  - The proof of optimality can be given within six minutes.
  - Making the formulation suitable for the monthly campaign preparation planning.

- **National Scale (Time-Limited Applicability:**

For scenarios with $D \geq 6$ and $S \geq 40$, together with $I = 30$ and $T = 12$, similar behavioral patterns were observed.

  - Solution times rise but still deliver ≤0.1% gap within 1–3 hours,
  - Aligning with enterprise planning cycles and enable near-optimal policy updates.

- **Numerous Portfolios (Decomposition Advisable):**

For configurations beyond these levels (for example, D is greater than or equal to 8, S is greater than or equal to 80, I is greater than or equal to 50, and T is greater than or equal to 18):

  - Decomposition or hybrid heuristics become advisable to maintain responsiveness while preserving solution quality.

5.4 Positioning of the Findings in the Literature

The empirical findings extend prior omnichannel optimization research by demonstrating that a unified MILP can simultaneously capture:

- allocation decisions,
- inter-DC balancing, and
- channel-specific fulfillment flows

at realistic scales.

Unlike earlier studies that modeled pricing, allocation, or fulfillment independently, the integrated structure here reveals cross-channel substitution, seasonal dynamics, and consolidation behaviors that emerge endogenously from cost and demand interactions. The robustness of these patterns across network sizes highlights the contribution of this formulation to scalable, profit-oriented omnichannel planning.

6. Conclusion and Future Research

This study presents a mixed-integer linear programming (MILP) model that jointly optimizes product–channel allocation, inventory positioning, customer order fulfillment, and inter-channel transfer decisions within a multi-period omnichannel distribution network. The model integrates a wide range of operational components, including initial procurement costs, inventory holding costs, customer delivery expenses, and inter-channel transfer costs, into a framework focused on maximizing profits, effectively merging multiple decision layers that are typically addressed independently in existing research.

The empirical application, conducted using operational data from a real cosmetics company, demonstrates that the model is capable of accurately reproducing the firm's historical flow patterns. Our results show that the proposed formulation successfully captures the operational logic of the system and can function as a practical decision-support mechanism.

In terms of computational performance, the results indicate that the model exhibits a scalable structure. While regional and medium-sized instances yield optimal solutions within minutes, even large-scale national scenarios are solved within reasonable timeframes with acceptable MIP gaps. This level of performance suggests that the model aligns well with the S&OP planning horizons and is suitable for use in operational planning processes.

Although the study provides an integrated framework, several limitations create opportunities for future research. The model operates under deterministic demand and fixed parameters, whereas real omnichannel environments can exhibit uncertainty, variable lead times, promotional dynamics, and varying service-level requirements. Similarly, binding constraints such as warehouse capacity, labor limitations, or transportation capacity were not incorporated into the current version. Additionally, the model has been validated within a single sector and country context; other industries may involve more complex channel behaviors or stronger price sensitivity.

The potential directions for future work include the following:

(1) Incorporating uncertainty and service-level requirements: Integrating demand uncertainty, variable lead times, and probabilistic service levels would enable more robust planning structures.

(2) Introducing capacity constraints: Adding constraints related to storage space, labor availability, picking stations, and transportation capacity would produce more realistic results, particularly during intensive promotional periods.

(3) Advanced decomposition and acceleration techniques for large-scale networks: Lagrangian relaxation and Benders decomposition accelerate the solution of large-scale network problems by directly applying advanced decomposition and acceleration techniques. For numerous instances where full solvers become computationally demanding, heuristic or approximation methods could provide practical near-optimal solutions.

(4) Rolling-horizon planning: Integrating rolling-horizon updates would create a more dynamic structure aligned with real-time operational planning practices.

(5) Behavioral components and cross-sector validation: Incorporating channel migration, pricing effects, or demand-shaping dynamics, along with empirical applications in different sectors, would improve the model's generalizability.

This study introduces a comprehensive, profit-driven, and evidence-based decision model for omnichannel distribution networks. The proposed framework addresses a major gap in the existing literature and offers a robust foundation for both theoretical analysis and real-world application.

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